

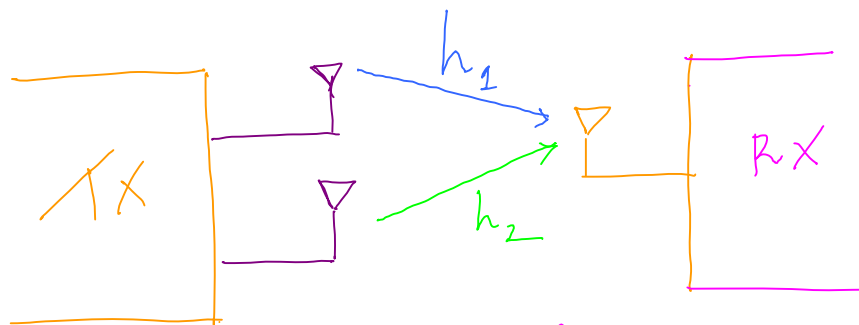
MISO



Multiple Input
Single Output Systems

Multiple TX antennas
Single RX antenna

Transmit Beamforming



2 Transmit antennas

1 Receive antenna

The model for 1x2 MISO system

$$\underbrace{y}_{\substack{1 \times 1 \\ 1 \times 1}} = \underbrace{[h_1 \ h_2]}_{1 \times 2} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\substack{2 \times 1}} + \underbrace{w}_{\substack{1 \times 1 \\ \text{Gaussian} \\ \text{mean } 0 \\ \text{var } \sigma^2}}$$

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Symbol on Tx 1

Symbol on Tx 2

What is the Optimal Transmission Scheme?

Let x denote a Transmit symbol.

$$\bar{x} = \frac{1}{\|h\|} \begin{bmatrix} h_1^* \\ h_2^* \end{bmatrix} x$$

Transmit vector (pointing to $\bar{x}$)
 Transmit Beamformer (pointing to the matrix)
 Transmit Symbol (pointing to x)

$$\|h\| = \sqrt{|h_1|^2 + |h_2|^2}$$

$$\text{Transmit Beamformer} = \frac{1}{\|h\|} \begin{bmatrix} h_1^* \\ h_2^* \end{bmatrix}$$

Unit Vector (pointing to the matrix)

$$\bar{x} = \begin{bmatrix} \frac{h_1^*}{\|h\|} x \\ \frac{h_2^*}{\|h\|} x \end{bmatrix}$$

From TX 1 : $\frac{h_1^* x}{\|h\|}$

From TX 2 : $\frac{h_2^* x}{\|h\|}$

$$y = [h_1 \quad h_2] \begin{bmatrix} h_1^* \\ h_2^* \end{bmatrix} \frac{1}{\|h\|} x + w$$

$$y = \|h\| x + w$$

$$SNR = \frac{\|h\|^2 E\{|x|^2\}}{\sigma^2}$$

$$= \frac{\|h\|^2 P}{\sigma^2}$$

} Same SNR as MRC or Maximal Ratio Combining.

Transmit Beamforming is able to achieve same SNR as MRC.

$$\text{Transmit Vector} = \begin{bmatrix} h_1^* \\ h_2^* \end{bmatrix} \frac{1}{\|h\|} x$$

Channel coefficients or Channel State is to be known at Transmitter
CSI - Channel State Information

For Tx Beamforming, CSI
is required at Transmitter



CHALLENGING.

Example:

Consider a 1×2 MISO system

$$y = \begin{bmatrix} \underbrace{2+j}_{h_1} & \underbrace{1-2j}_{h_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + w$$

$$h_1 = 2+j$$

$$h_2 = 1-2j$$

What is Optimal Transmit Beamforming
if CSI is available at Tx?

$$\bar{x} = \begin{bmatrix} h_1^* \\ h_2^* \end{bmatrix} \frac{1}{\|h\|} \cdot x$$

$$h_1 = 2 + j \quad h_2 = 1 - 2j$$

$$\|h\| = \sqrt{4 + 1 + 1 + 4}$$

$$= \sqrt{10}$$

$$\bar{x} = \frac{1}{\sqrt{10}} \begin{bmatrix} 2-j \\ 1-2j \end{bmatrix} x$$

$$y = \begin{bmatrix} 2+j & 1+2j \end{bmatrix} \begin{bmatrix} 2-j \\ 1-2j \end{bmatrix} \frac{1}{\sqrt{10}} x + w$$

$$= \sqrt{10} x + w$$

$\searrow$
 $\|h\|$

$$\text{SNR} = 10 \frac{P}{\sigma^2} = \frac{\|h\|^2 P}{\sigma^2}$$

SNR with
maximum ratio
Combining

BER of Transmit Beamforming:

Transmit beamforming can be extended to a general scenario with L antennas.

$$y = [h_1 \ h_2 \ \dots \ h_L] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_L \end{bmatrix} + w$$
$$= \mathbf{h}^T \bar{\mathbf{x}} + w$$

$$y = \underbrace{\mathbf{h}^T}_{1 \times L} \underbrace{\bar{x}}_{L \times 1} + w \leftarrow |X| \text{ Gaussian mean } 0 \text{ var } \sigma^2$$

$$\bar{x} = \frac{1}{\|\mathbf{h}\|} \cdot \mathbf{h}^* \cdot x$$

Transmit Symbol.

$$y = \mathbf{h}^T \cdot \frac{\mathbf{h}^*}{\|\mathbf{h}\|} x + w$$

$$= \|\mathbf{h}\| x + w$$

$$\text{SNR} = \frac{\|\mathbf{h}\|^2 P}{\sigma^2}$$

Equal to MRC

Therefore BER of Tx
Beamforming is identical to
that of MRC.

At high SNR,

$$\text{BER} = 2^{L-1} Q \left(\frac{1}{2 \text{SNR}} \right)^L$$

L = Number
of TX antennas

Example: Consider 1×3 MISO system. $\text{SNR} = 25 \text{ dB}$.

Average BER = ?

$$\text{BER} = 2^{L-1} C_L \cdot \left(\frac{1}{2 \text{SNR}} \right)^3$$

$$L = 3$$

$$2^{L-1} C_L = {}^5 C_3 = 10$$

$$\text{BER} = \frac{10}{8} \cdot \frac{1}{\text{SNR}^3}$$

$$\text{SNR}(\text{dB}) = 25 \text{ dB}$$

$$10 \log_{10} \text{SNR} = 25$$

$$\text{SNR} = 10^{2.5}$$

$$\begin{aligned} \text{BER} &= \frac{10}{8} \cdot \frac{1}{(10^{2.5})^3} \\ &= 6.3 \times 10^{-8} \end{aligned}$$
